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STUDY OF SEMI-INVARIANT SUBMANIFOLDS OF A NEARLY TRANS-SASAKIAN MANIFOLD ADMITTING A SEMI-SYMMETRIC NON-METRIC CONNECTION

MOBIN AHMAD, JAE-BOK JUN AND MOHD DANISH SIDDIQI

Abstract. In this paper a semi-symmetric non-metric connection in a nearly trans-Sasakian manifold is defined and semi-invariant submanifolds of a nearly trans-Sasakian manifold endowed with a semi-symmetric non-metric connection is studied. Moreover, Nijenhuis tensor is calculated and integrability conditions of the distributions on semi-invariant submanifolds are discussed.

1. Introduction

The study of differential geometry of semi-invariant or contact CR-submanifolds, as a generalization of invariant and anti-invariant submanifolds, of an almost contact metric manifold was initiated by Bejancu ([4],[5]) and was followed by Kobayshi [11], Yano [17] and other Geometers (cf. [16], [14], [7] etc.). One has also the notion of α-Sasakian and β-Kenmotsu structures [10]. Oubina introduced a new class of almost contact Riemannian manifolds known as trans-Sasakian manifolds [15]. Both α-Sasakian and β-Kenmotsu structures belong to this new class. C. Gherghe [9] introduced a nearly trans-Sasakian structure of type (α, β), which generalizes tran-Sasakian structure in the same sense as nearly Sasakian generalizes Sasakian one. A trans-Sasakian structure is always a nearly trans-Sasakian structure. Moreover, nearly trans-Sasakian structure of type (α, β) is nearly-Sasakian ([2], [7]) or nearly Kenmotsu [1] or nearly cosymplectic [6] accordingly as $\beta = 0$ or $\alpha = 0$; or $\alpha = \beta = 0$. Matsumoto et al. in [14] studied semi-invariant submanifold of a trans-Sasakian manifolds and after that Jeong-Sik Kim and others studied the semi-invariant submanifolds of nearly trans-Sasakian manifolds [12]. In this paper, we study semi-invariant submanifolds of nearly trans-Sasakian manifolds with semi-symmetric non-metric connection.

In 1924, A. Friedmann and J.A. Schouten [8] introduced the idea of a semi-symmetric linear connection. A linear connection ∇ is said to be a *semi-symmetric* connection if its torsion tensor T is of the form

$$T(X,Y) = \eta(Y)X - \eta(X)Y,$$

where η is a 1-form.

The connection ∇ is *symmetric* if the torsion tensor T vanishes, otherwise, it is *non-symmetric*. The connection ∇ is *metric* connection if there is a Riemannian

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metric g in M such that $\nabla g = 0$, otherwise it is *non-metric*. It is well known that a linear connection is symmetric and metric if and only if it is the Levi-Civita connection. Some properties of semi-invariant submanifolds, hypersurfaces and submanifolds with semi-symmetric non-metric connection were studied in ([1], [2], [3]) respectively.

In this paper, we study semi-invariant submanifolds of nearly trans-Sasakian manifolds with a semi-symmetric non-metric connection. This paper is organized as follows. In section 2, we give a brief introduction of nearly trans-Sasakian manifold. In section 3, we recall some necessary details about semi-invariant submanifolds. In section 4, we derive Nijenhuis tensor for nearly trans-Sasakian manifold with semi-symmetric non-metric connection. In section 5, some basic results on nearly trans-Sasakian manifold with semi-symmetric non-metric connection are obtained. In section 6, 7 and 8, integrability of some distributions on nearly trans-Sasakian manifold are discussed.

2. Nearly trans-Sasakian manifold

Let $\bar{M}$ be an almost contact metric manifold [6] with an almost contact metric structure (ϕ, ξ, η, g), where ϕ is a (1,1)-tensor field, ξ is a vector field, η is a 1-form and g is a compatible Riemannian metric such that

$$\begin{aligned} &\phi^2 = -I + \eta\otimes\xi, \quad \phi\xi = 0, \quad \eta o\phi = 0, \quad \eta(\xi) = 1 \\ &g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \\ &g(X, \phi Y) = -g(\phi X, Y), \;\; g(X, \xi) = \eta(X) \end{aligned} \tag{2.1}$$

for all vector fields X, Y on $T\bar{M}$. There are two known classes of almost contact metric manifold, namely Sasakian and Kenmotsu manifolds. Sasakian manifolds are characterized by the tensorial relation

$$(\bar{\bar{\nabla}}_X\phi)Y = g(X, Y)\xi - \eta(Y)X,$$

while Kenmotsu manifolds are given by the tensor equation

$$(\bar{\bar{\nabla}}_X\phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X.$$

An almost contact metric structure (ϕ, ξ, η, g) on $\bar{M}$ is called a *trans-Sasakian* structure [15] if $(\bar{M} \times R, J, G)$ belongs to the class W_4 of the Gray-Hervella classification of almost Hermitian manifolds [9], where J is the almost complex structure on $\bar{M} \times R$ defined by

$$J(X, ad/dt) = (\phi X - a\xi, \eta(X)d/dt)$$

for all vector fields X on $\bar{M}$ and smooth function a on $\bar{M} \times R$ and G is the product metric on $\bar{M} \times R$. This may be expressed by the condition [13]

$$(\bar{\bar{\nabla}}_X\phi)Y = \alpha(g(X, Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi - \eta(Y)\phi X) \tag{2.2}$$

for some smooth functions α and β on $\bar{M}$ and we say that the trans-Sasakian structure is of type (α, β).

The class $C_6 \oplus C_5$ [15] coincides with the class of trans-Sasakian structures of type (α, β). We note that trans-Sasakian structures of type (0,0) are cosymplectic [3], trans-Sasakian structures of tpye $(\alpha, 0)$ are α-Sasakian [10]. Recently, C. Gherghe [9] introduced a nearly trans-Sasakian structure of type (α, β). An almost contact metric structure (ϕ, ξ, η, g) on $\bar{M}$ is called a *nearly trans-Sasakian* structure ([2], [7]) if

$$(2.3)\quad (\bar{\bar{\nabla}}_X\phi)Y + (\bar{\bar{\nabla}}_Y\phi)X = \alpha(2g(X,Y)\xi - \eta(Y)X - \eta(X)Y) - \beta(\eta(Y)\phi X + \eta(X)\phi Y).$$

A trans-Sasakian structure is always a nearly trans-Sasakian structure. Moreover a nearly trans-Sasakian structure of type (α, β) is nearly-Sasakian [15] or nearly Kenmotsu [1] or nearly cosymplectic [6] accordingly as $\beta = 0$, $\alpha = 1$; or $\alpha = 0, \beta = 1$; or $\alpha = 0, \beta = 0$ respectively.

Owing to the existence of 1-form η, we can define a semi-symmetric non-metric connection in contact manifold by

$$(2.4)\qquad \bar{\nabla}_X Y = \bar{\bar{\nabla}}_X Y + \eta(Y)X$$

for all $X, Y \in TM$. From (2.3) and (2.4), we have

$$(2.5)\quad (\bar{\nabla}_X\phi)Y + (\bar{\nabla}_Y\phi)X = \alpha(2g(X,Y)\xi - \eta(Y)X - \eta(X)Y) - (\beta+1)(\eta(Y)\phi X + \eta(X)\phi Y).$$

A trans-Sasakian structure is always a nearly trans-Sasakian structure. Moreover, a nearly trans-Sasakian structure with semi-symmetric non-metric connection of type (α, β) is nearly Sasakian [15] or nearly Kenmotsu [1] or nearly cosympletic [3] accordingly as $\beta = 0, \alpha = 1$; or $\alpha = 0, \beta = 1$; or $\alpha = 0, \beta = 0$ respectively.

A nearly trans-Sasakian structure of type (α, β) will be called nearly α-Sasakian (resp. nearly β-Kenmotsu) if β=0 (resp. $\alpha = 0$).

Thus the structural equations for nearly α-Sasakian, nearly Sasakian, nearly β-Kenmotsu, nearly Kenmotsu and nearly cosympletic manifolds are respectively given by

$$(2.6)\quad \begin{aligned}
(\bar{\nabla}_X\phi)Y + (\bar{\nabla}_Y\phi)X =& \alpha(2g(X,Y)\xi - \eta(Y)X - \eta(X)Y) \\
& - (\eta(Y)\phi X + \eta(X)\phi Y), \\
(\bar{\nabla}_X\phi)Y + (\bar{\nabla}_Y\phi)X =& (2g(X,Y)\xi - \eta(X)Y - \eta(Y)X) \\
& - (\eta(Y)\phi X + \eta(X)\phi Y), \\
(\bar{\nabla}_X\phi)Y + (\bar{\nabla}_Y\phi)X =& - (\beta+1)(\eta(Y)\phi X + \eta(X)\phi Y), \\
(\bar{\nabla}_X\phi)Y + (\bar{\nabla}_Y\phi)X =& - 2(\eta(Y)\phi X + \eta(X)\phi Y), \\
(\bar{\nabla}_X\phi)Y + (\bar{\nabla}_Y\phi)X =& - (\eta(Y)\phi X + \eta(X)\phi Y).
\end{aligned}$$

3. **Semi-invariant submanifolds**

Let M be a submanifold of a Riemannian manifold $\bar{M}$ with Riemannian metric g. Then Gauss and Weingarten formulae are given respectively by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X,Y),\ (X, Y \in TM),$$

$$\bar{\nabla}_X N = -A_N X + \nabla^{\perp}_X N + \eta(N)X,\ (N \in T^{\perp}M),$$

where $\bar{\nabla}$, ∇ and $\nabla^{\perp}$ are the semi-symmetric non-metric connection, induced connection and induced normal connection in $\bar{M}$, M and the normal bundle $T^{\perp}M$ of M respectively and h is the second fundamental form related to A by

$$g(h(X,Y), N) = g(A_N X, Y).$$

Moreover, if ϕ is a $(1,1)$-tensor field on $\bar{M}$, we have for $X \in TM$ and $N \in T^{\perp}M$

$$
\begin{aligned}
(\bar{\nabla}_X\phi)Y =&(\nabla_X P)Y - A_{FY}X - th(X,Y)\\
&+ (\nabla_X F)Y + h(X,PY) - fh(X,Y) + \eta(FY)X,\\
(\bar{\nabla}_X\phi)N =&(\nabla_X t)N - A_{fN}X - PA_N X - \eta(N)PX\\
&+ (\nabla_X f)N + h(X,tN) - FA_N X - \eta(N)FX + \eta(fN)X,
\end{aligned} \tag{3.1}
$$

where

$$
\begin{aligned}
&\phi X = PX + FX, \quad (PX \in TM, FX \in T^{\perp}M),\\
&\phi N = tN + fN, \quad (tN \in TM, fN \in T^{\perp}M),\\
&(\nabla_X P)Y = \nabla_X(PY) - P\nabla_X Y, \quad (\nabla_X F)Y = \nabla^{\perp}_X(FY) - F\nabla_X Y,\\
&(\nabla_X t)N = \nabla_X(tN) - t\nabla^{\perp}_X N, \quad (\nabla_X f)N = \nabla^{\perp}_X(fN) - f\nabla^{\perp}_X N.
\end{aligned} \tag{3.2}
$$

The submanifold M is totally geodesic if $h = 0$, minimal if $H = trace(h)/dim(M) = 0$ and totally umbilical if $h(X,Y) = g(X,Y)H$ in $\bar{M}$ respectively.

For a distribution D on M, M is said to be *D-totally geodesic* if for all $X, Y \in D$, we have $h(X,Y) = 0$. If for all $X, Y \in D$, we have $h(X,Y) = g(X,Y)K$ for some normal vector K, then M is called *D-totally umbilical.* For two distributions D and $\mathcal{E}$ defined on M, M is said to be *$(D, \mathcal{E})$-mixed totally geodesic* if for all $X \in D$ and $Y \in \mathcal{E}$ we have $h(X,Y) = 0$.

Let D and $\mathcal{E}$ be two distributions defined on a manifold M. We say that D is *$\mathcal{E}$-parallel* if for all $X \in \mathcal{E}$ and $Y \in D$, we have $\nabla_X Y \in D$. If D is D-parallel, then it is called *autoparallel*. D is called *X-parallel* for some $X \in TM$ if for all $Y \in D$ we have $\nabla_X Y \in D$. D is said to be *parallel* if for all $X \in TM$ and $Y \in D$, $\nabla_X Y \in D$. If a distribution D on M is autoparallel, then it is clearly integrable and by Gauss formula D is totally geodesic in M. If D is parallel, then orthogonal complementary distribution $D^{\perp}$ is also parallel, which implies that D is parallel if and only if $D^{\perp}$ is parallel. In this case M is locally the product of the leaves of D and $D^{\perp}$.

Let M be a submanifold of an almost contact metric manifold. If $\xi \in TM$, then we write $TM = \{\xi\} \oplus \{\xi\}^{\perp}$, where $\{\xi\}$ is the distribution spanned by ξ and $\{\xi\}^{\perp}$ is the complementary orthogonal distribution of $\{\xi\}$ in M. Then we get

$$
\begin{aligned}
&P\xi = 0 = F\xi, \quad \eta o P = 0 = \eta o F,\\
&P^2 + tF = -I + \eta \otimes \xi, \quad FP + fF = 0,\\
&f^2 + Ft = -I, \quad tf + Pt = 0.
\end{aligned} \tag{3.3}
$$

A submanifold M of an almost contact metric manifold $\bar{M}$ with $\xi \in TM$ is called *semi-invariant* submanifold [2] of $\bar{M}$ if there exists two differentiable distributions D^1 and D^0 on M such that

(i) $TM = D^1 \oplus D^0 \oplus \{\xi\}$,
(ii) the distribution D^1 is invariant by ϕ, that is $\phi(D^1) = D^1$ and
(iii) the distribution D^0 is anti – invariant by ϕ, that is $\phi(D^0) \subseteq T^{\perp}M$.

For $X \in TM$, we can write

$$X = U^1 X + U^0 X + \eta(X)\xi,$$

where U^1 and U^0 are the projection operators of TM on D^1 and D^0 respectively. A semi-invariant submanifold of an almost contact metric manifold becomes an invariant submanifold ([2], [3]) (resp. anti-invariant submanifold ([2], [3])) if $D^0 = \{0\}$ (resp. $D^1 = \{0\}$).

4. Nijenhuis tensor

An almost contact metric manifold is said to be *normal* [3] if the torsion tensor $N^{(1)}$ vanishes, that is:

$$N^{(1)} \equiv [\phi, \phi] + 2d\eta \otimes \xi = 0, \tag{4.1}$$

where $[\phi, \phi]$ is the Nijenhuis tensor of ϕ and d denotes the exterior derivative operator.

In this section we obtain expression for Nijenhuis tensor $[\phi, \phi]$ of the structure tensor field ϕ given by $[\phi, \phi](X, Y) = ((\bar{\nabla}_{\phi X}\phi)Y - (\bar{\nabla}_{\phi Y}\phi)X) - \phi((\bar{\nabla}_X\phi)Y - (\bar{\nabla}_Y\phi)X)$ in a nearly trans-Sasakian manifold. In particular, we derive the expressions for the Nijenhuis tensor $[\phi, \phi]$ in nearly Sasakian manifold and nearly Kenmotsu manifolds.

First, we need the following Lemma.

Lemma 4.1. *In an almost contact metric manifold we have*

$$(\bar{\nabla}_Y\phi)(\phi X) = -(\phi(\bar{\nabla}_Y\phi)X + (\bar{\nabla}_Y\eta)X)\xi + \eta(X)\bar{\nabla}_Y\xi. \tag{4.2}$$

Proof. For $X, Y \in TM$, we have

$$\begin{aligned}(\bar{\nabla}_Y\phi)(\phi X) &= \bar{\nabla}_Y(\phi^2 X) - \phi(\bar{\nabla}_Y\phi X) + \phi(\phi\bar{\nabla}_Y X) - \phi^2\bar{\nabla}_Y X \\ &= \bar{\nabla}_Y(-X + \eta(X)\xi) - \phi(\bar{\nabla}_Y\phi X) \\ &\quad + \phi(\phi\bar{\nabla}_Y X) - (-\bar{\nabla}_Y X + \eta(\bar{\nabla}_Y X)\xi),\end{aligned}$$

which gives the equation (4.2). □

Now, we prove the following theorem.

Theorem 4.2. *In a nearly trans-Sasakian manifold with semi-symmetric non-metric connection, the Nijenhuis tensor $[\phi, \phi]$ of ϕ is given by*

$$\begin{aligned}[\phi, \phi](X, Y) =& 4\phi(\bar{\nabla}_Y\phi)X + 2d\eta(X, Y)\xi - \eta(X)\bar{\nabla}_Y\xi + \eta(Y)\bar{\nabla}_X\xi \\ &+ \alpha(\eta(Y)\phi X + 3\eta(X)\phi Y) + 4g(\phi X, Y)\xi \\ &+ 4(\beta + 1)(\eta(X)\eta(Y)\xi + (\beta + 1)(-\eta(Y)X - 3\eta(X)Y).\end{aligned} \tag{4.3}$$

Proof. Using Lemma 4.1 and $\eta o \phi = 0$ in (2.3) we get

$$\begin{aligned}(\bar{\nabla}_{\phi X}\phi)Y =& \phi(\bar{\nabla}_Y\phi)X - ((\bar{\nabla}_Y\eta)X)\xi - \eta(X)\bar{\nabla}_Y\xi \\ &+ \alpha(2g(\phi X, Y)\xi - \eta(Y)\phi X) \\ &- (\beta + 1)(-\eta(Y)X + \eta(Y)\eta(X)\xi).\end{aligned} \tag{4.4}$$

Thus we get

$$
\begin{aligned}
[\phi,\phi](X,Y) =&((\bar{\nabla}_{\phi X}\phi)Y + \phi((\bar{\nabla}_Y\phi)X)) - ((\bar{\nabla}_{\phi Y}\phi)X + \phi(\bar{\nabla}_X\phi)Y)\\
=&2\phi(\bar{\nabla}_Y\phi)X - ((\bar{\nabla}_Y\eta)X)\xi - \eta(X)\bar{\nabla}_Y\xi\\
&+ ((\bar{\nabla}_X\eta)Y)\xi + \eta(Y)\bar{\nabla}_X\xi - \alpha(2g(\phi Y,X)\xi - \eta(Y)\phi X)\\
&- (\beta+1)(-\eta(Y)X + \eta(Y)\eta(X)\xi) - 2\phi(\bar{\nabla}_X\phi)Y\\
&- (\beta+1)(-\eta(X)Y + \eta(X)\eta(Y)\xi)\\
&+ \alpha(2g(\phi X,Y)\xi - \eta(Y)\phi X)\\
=&2\phi((\bar{\nabla}_Y\phi)X - (\bar{\nabla}_X\phi)Y) + 2d\eta(X,Y)\xi - \eta(X)\bar{\nabla}_Y\xi + \eta(Y)\bar{\nabla}_X\xi\\
&- (\beta+1)(-\eta(X)Y - \eta(Y)X) + 2(\beta+1)\eta(X)\eta(Y)\xi\\
&+ \alpha(4g(\phi X,Y)\xi - \eta(Y)\phi X + \eta(X)\phi Y)\\
=&2\phi((\bar{\nabla}_Y\phi)X + (\bar{\nabla}_Y\phi)X) - 2\alpha\phi(2g(X,Y)\xi - \eta(Y)X - \eta(X)Y)\xi\\
&+ \alpha(4g(\phi X,Y)\xi + \eta(Y)\phi X + \eta(X)\phi Y) - (\beta+1)(-\eta(Y)\phi X - \eta(X)\phi Y)\\
&- \eta(X)\bar{\nabla}_Y\xi + \eta(Y)\bar{\nabla}_X\xi - (\beta+1)(-\eta(X)Y - \eta(Y)X)\\
&+ 2(\beta+1)\eta(X)\eta(Y)\xi + 2d\eta(X,Y)\\
=&4\phi(\bar{\nabla}_Y\phi)X - \eta(X)\bar{\nabla}_Y\xi + \eta(Y)\bar{\nabla}_X\xi\\
&+ \alpha(\eta(Y)\phi X + 3\eta(X)\phi Y + 4g(\phi X,Y)\xi)\\
&+ 2d\eta(X,Y)\xi - (\beta+1)(\eta(Y)\phi^2X + 3\eta(X)\phi^2Y),
\end{aligned}
$$

which implies the equation(4.3). □

From equation (4.3), we get

$$
\eta(N^1(X,Y)) = 4d\eta(X,Y) - 4\alpha g(X,\phi Y). \tag{4.5}
$$

Corollary 4.3. *In a nearly Sasakian manifold with semi-symmetric non-metric connnection, the Nijenhuis tensor $[\phi,\phi]$ of ϕ is given by*

$$
\begin{aligned}
[\phi,\phi](X,Y) =&4\phi(\bar{\nabla}_Y\phi)X + 2d\eta(X,Y)\xi - \eta(X)\bar{\nabla}_Y\xi + \eta(Y)\bar{\nabla}_X\xi\\
&- \eta(X)Y - 3\eta(Y)X + \eta(Y)\phi X + 3\eta(X)\phi Y\\
&- 4g(X,\phi Y)\xi - 4\eta(X)\eta(Y)\xi.
\end{aligned}
$$

Consequently,

$$
\eta(N^1(X,Y)) = 4d\eta(X,Y) - 4g(X,\phi Y) - 4\eta(X)\eta(Y).
$$

In particular, if X and Y are perpendicular to ξ, then

$$
\begin{aligned}
[\phi,\phi](X,Y) =&4\phi(\bar{\nabla}_Y\phi)X - 2\eta([X,Y])\xi - 4g(X,\phi Y)\xi\\
&- 4\eta(X)\eta(Y)\xi.
\end{aligned}
$$

Corollary 4.4. *In a nearly Kenmotsu manifold with semi-symmetric non-metric connection, the Nijenhuis tensor $[\phi,\phi]$ of ϕ is given by*

$$
\begin{aligned}
[\phi,\phi](X,Y) =&4\phi(\bar{\nabla}_Y\phi)X + 2d\eta(X,Y)\xi - \eta(X)\bar{\nabla}_Y\xi + \eta(Y)\bar{\nabla}_X\xi\\
&+ 8\eta(X)\eta(Y)\xi - 2(\eta(Y)\phi X + 3\eta(X)\phi Y).
\end{aligned}
$$

Consequently,

$$
\eta(N^1(X,Y)) = 4d\eta(X,Y).
$$

In particular, if X and Y are perpendicular to ξ, then

$$[\phi,\phi](X,Y) = 4\phi(\bar{\nabla}_Y\phi)X - 2\eta([X,Y])\xi.$$

5. Some basic results

Let M be a submanifold of a nearly trans-Sasakian manifold. Using (3.1), (3.2) in (2.6) for $X, Y \in TM$, we get

$$\begin{aligned}\alpha(2g(X,Y)\xi-&\eta(X)Y - \eta(Y)X) - (\beta+1)(\eta(Y)PX \\ &+\eta(Y)FX + \eta(X)PY + \eta(X)FY) + 2\eta(X)\eta(Y)\xi \\ &=(\nabla_X P)Y + (\nabla_Y P)X - A_{FY}X - A_{FX}Y - 2th(X,Y) - 2fh(X,Y) \\ &\quad + (\nabla_X F)Y + (\nabla_Y F)X + h(X,PY) + h(PX,Y) \\ &\quad + \eta(FY)X + \eta(FX)Y.\end{aligned}$$

Thus, we have the following.

Proposition 5.1. *Let M be a submanifold of a nearly trans-Sasakian manifold with semi-symmetric non-metric connection. Then for all $X, Y \in TM$, we have*

$$\begin{aligned}(5.1)\quad &(\nabla_X P)Y + (\nabla_Y P)X - A_{FY}X - A_{FX}Y - 2th(X,Y) \\ &= \alpha(2g(X,Y)\xi - \eta(X)Y - \eta(Y)X) - (\beta+1)(\eta(Y)PX + +\eta(X)PY)\end{aligned}$$

and

$$\begin{aligned}(5.2)\quad &(\nabla_X F)Y + (\nabla_Y F)X + h(X,PY) + h(PX,Y) - 2fh(X,Y) \\ &= -(\beta+1)(\eta(Y)FX + \eta(X)FY) - \eta(FX)Y - \eta(FY)X.\end{aligned}$$

Now, we state the following proposition.

Proposition 5.2. *Let M be a submanifold of a nearly trans-Sasakian manifold with semi-symmetric non-metric connection. Then*

$$\begin{aligned}\bar{\nabla}_X\phi Y - \bar{\nabla}_Y\phi X - \phi[X,Y] =&2((\nabla_X P)Y - A_{FY}X - th(X,Y) + \eta(FY)X) \\ &+2((\nabla_X F)Y + h(X,PY) - fh(X,Y) + \eta(FX)Y) \\ &+\alpha(\eta(Y)X + \eta(X)Y - 2g(X,Y)\xi) \\ &+(\beta+1)(\eta(Y)PX + \eta(X)PY) \\ &+(\beta+1)(\eta(Y)FX + \eta(X)FY).\end{aligned}$$

Consequently,

$$\begin{aligned}P[X,Y] =& -\nabla_X PY - \nabla_Y PX + A_{FX}Y + A_{FY}X + 2P\nabla_X Y + 2th(X,Y) \\ &+ \alpha(\eta(Y)X + \eta(X)Y - 2g(X,Y)\xi) \\ &- (\beta+1)(\eta(Y)PX + \eta(X)PY),\end{aligned}$$

and

$$\begin{aligned}(5.3)\quad F[X,Y] =& -\nabla^{\perp}_X FY - \nabla^{\perp}_Y FX - h(X,PY) - h(PX,Y) + 2F\nabla_X Y \\ &+ 2fh(X,Y) - (\beta+1)(\eta(Y)FX + \eta(X)FY) - \eta(FY)X + \eta(FX)Y\end{aligned}$$

for all $X, Y \in TM$. The proof is straightforward and hence omitted.

Proposition 5.3. *Let M be a semi-invariant submanifold of a nearly trans-Sasakian manifold with semi-symmetric non-metric connection. Then (P, ξ, η, g) is a nearly trans-Sasakian structure on the distribution $D^1 \oplus \{\xi\}$ if $th(X,Y) = 0$ for all $X, Y \in D^1 \oplus \{\xi\}$.*

Proof. From $D^1 \oplus \{\xi\} = Ker(F)$ and (3.3) we have $P^2 = -I + \eta \otimes \xi$ on $D^1 \oplus \{\xi\}$. We also get $P\xi = 0, \eta(\xi) = 1, \eta o P = 0$. Using $D^1 \oplus \{\xi\} = Ker(F)$ and $th(X,Y) = 0$ in (5.1) we get

$$\begin{aligned}(\nabla_X P)Y + (\nabla_Y P)X = &- (\beta+1)(\eta(Y)PX + \eta(X)PY)\\ &+ \alpha(2g(X,Y)\xi - \eta(X)Y - \eta(Y)X),\end{aligned}$$

where $X, Y \in D^1 \oplus \{\xi\}$. This completes the proof. □

Theorem 5.4. *Let M be a semi-invariant submanifold of a nearly trans-Sasakian manifold with semi-symmetric non-metric connection. Then we have*

(i) if $D^0 \oplus \{\xi\}$ is autoparallel, then

$$A_{FX}Y + A_{FY}X + 2th(X,Y) = 0, \;\; X, Y \in D^0 \oplus \{\xi\},$$

(ii) if $D^1 \oplus \{\xi\}$ is autoparallel, then

$$h(X, PY) + h(PX, Y) - 2fh(X,Y) = 0, \;\; X, Y \in D^1 \oplus \{\xi\}.$$

Proof. In view of (5.1) and autoparallelness of $D^0 \oplus \{\xi\}$ we get (i), while in view of (5.2) and appropriateness of $D^1 \oplus \{\xi\}$, we get (ii). □

In view of Proposition 5.3 and (ii) (Theorem 5.4), we get

Theorem 5.5. *Let M be a submanifold of nearly trans-Sasakian manifold with semi-symmetric non-metric connection with $\xi \in TM$. If M is invariant, then M is nearly trans-Sasakian. Moreover,*

$$h(X, PY) + h(PX, Y) - 2fh(X,Y) = 0$$

for all $X, Y \in TM$.

6. Integrability of the distribution $\mathbf{D^1} \oplus \{\xi\}$

We begin with the following lemma.

Lemma 6.1. *Let M be a semi-invariant submanifold of a nearly trans-Sasakian manifold with semi-symmetric non-metric connection for $X, Y \in D^1 \oplus \{\xi\}$. Then we get*

$$F[X,Y] = -h(X, PY) - h(PX, Y) + 2F\nabla_X Y + 2fh(X,Y) \tag{6.1}$$

or equivalently

$$-h(X, PX) + F\nabla_X X + fh(X,X) = 0$$

and

$$\eta(FX)Y = \eta(FY)X$$

for all $X, Y \in D^1 \oplus \{\xi\}$.

Proof. Equation (6.1) follows from $D^1 \oplus \{\xi\} = Ker(F)$ and (5.3) equivalence of (6.1) and $D^1 \oplus \{\xi\} = Ker(F)$. □

Thus we can state the following theorem.

Theorem 6.2. *The distribution $D^1 \oplus \{\xi\}$ on semi-invariant submanifold of a nearly trans-Sasakian manifold with semi symmetric non-metric connection is integrable if and only if*

$$h(X, PY) + h(PX, Y) = 2(F\nabla_X Y + fh(X, Y)).$$

Now, we need the following.

Definition 6.3. Let M be a Riemannian manifold with the Riemannian connection ∇. A distribution D on M is said to be *nearly autoparallel* if for all $X, Y \in D$ we have $(\nabla_X Y + \nabla_Y X) \in D$ or equivalently $\nabla_X X \in D$.

Thus we have the following flow chart:
Parallel $\Rightarrow$ Autoparallel $\Rightarrow$ Nearly autoparallel,
Parallel $\Rightarrow$ Integrable,
Autoparallel $\Rightarrow$ Integrable, and
Nearly autoparallel + Integrable $\Rightarrow$ Autoparallel.

Theorem 6.4. *Let M be a semi-invariant submanifold of a nearly trans-Sasakian manifold with semi-symmetric non-metric connection. Then following four statements*

(a) *the distribution $D^1 \oplus \{\xi\}$ is autoparallel,*
(b) *$h(X, PY) + h(PX, Y) = 2fh(X, Y)$ for $X, Y \in D^1 \oplus \{\xi\}$,*
(c) *$h(X, PX) = fh(X, X)$ for $X \in D^1 \oplus \{\xi\}$,*
(d) *the distribution $D^1 \oplus \{\xi\}$ is nearly autoparallel*

are related by (a) $\Rightarrow$ (b) $\Leftrightarrow$ (c) $\Rightarrow$ (d). In particular, if $D^1 \oplus \{\xi\}$ is integrable, then the above four statements are equivalent.

Let $X, Y \in D^1 \oplus \{\xi\}$. Using (2.1) and (3.6) in (4.1) we get

$$\begin{aligned} N^1(X, Y) = {} & 2d\eta(X, Y)\xi + [\phi X, \phi Y] - [X, Y] + \eta([X, Y])\xi \\ & - P([X, \phi Y] + [\phi X, Y]) - F([X, \phi Y] + [\phi X, Y]). \end{aligned} \tag{6.2}$$

On the other hand from equation (4.5), we have

$$\begin{aligned} (\bar\nabla_{\phi X}\phi)Y = {} & \phi(\bar\nabla_Y \phi)X - ((\bar\nabla_Y \eta)X)\xi - \eta(X)\bar\nabla_Y \xi + \alpha(2g(\phi X, Y)\xi - \eta(Y)\phi X) \\ & - (\beta + 1)(-\eta(Y)X + \eta(X)\eta(Y)\xi), \end{aligned}$$

which implies that

$$\begin{aligned} (\bar\nabla_{\phi X}\phi)Y - (\bar\nabla_{\phi Y}\phi)X = {} & \phi(\bar\nabla_Y \phi)X - (\bar\nabla_X \phi)Y + 2d\eta(X, Y)\xi \\ & - \eta(X)U^1\nabla_Y \xi - \eta(X)U^0\nabla_Y \xi + \eta(Y)U^1\nabla_X \xi \\ & + \eta(Y)U^0\nabla_X \xi - \eta(X)h(Y, \xi) + \eta(Y)h(X, \xi) \\ & + \alpha(\eta(X)\phi X - \eta(Y)\phi X) - (\beta + 1)(-\eta(X)Y - \eta(Y)X) \\ & - (\beta + 1)(2\eta(X)\eta(Y)\xi. \end{aligned} \tag{6.3}$$

Next, we can easily get

$$\begin{aligned} \phi(\bar\nabla_Y \phi)X & = \phi\bar\nabla_Y(\phi X) - \phi^2\bar\nabla_Y X \\ & = \phi(\nabla_Y(\phi X) + h(Y, \phi X)) + \bar\nabla_Y X - \eta(\bar\nabla_Y X)\xi, \end{aligned} \tag{6.4}$$

so that

$$\phi((\bar{\nabla}_Y\phi)X-(\bar{\nabla}_X\phi)Y)=-[X,Y]+\eta([X,Y])\xi+P(\nabla_Y\phi X-\nabla_X\phi Y)+F(\nabla_Y\phi X-\nabla_X\phi Y)+\phi(h(Y,\phi X)-h(X,\phi Y)). \tag{6.5}$$

In view of (6.3) and (6.5), we get

$$\begin{aligned}N^1(X,Y)=&-2[X,Y]+2P(\nabla_Y\phi X-\nabla_X\phi Y)+2F(\nabla_Y\phi X-\nabla_X\phi Y)\\&+2\phi(h(Y,\phi X)-h(X,\phi Y))-\eta(X)U^1\nabla_Y\xi-\eta(X)U^0\nabla_Y\xi\\&+\eta(Y)U^1\nabla_X\xi+\eta(Y)U^0\nabla_X\xi-\eta(X)h(Y,\xi)+\eta(Y)h(X,\xi)\\&+4d\eta(X,Y)\xi+\alpha(-\eta(X)\phi Y-\eta(Y)\phi X)\\&-(\beta+1)(-\eta(X)Y-\eta(Y)X)+2\eta([X,Y])\xi-2(\beta+1)\eta(X)\eta(Y)\xi.\end{aligned} \tag{6.6}$$

Theorem 6.5. *The distribution $D^1\oplus\{\xi\}$ is integrable on a semi-invariant submanifold M of nearly trans-Sasakian manifold with semi symmetric non-metric connection if and only if*

$$N^1(X,Y)\in D^1\oplus\{\xi\}, \tag{6.7}$$

$$\begin{aligned}2(h(Y,\phi X)-h(X,\phi Y))=&\,\eta(X)(\phi U^0\nabla_Y\xi+\alpha U^0Y+(\beta+1)\phi U^0Y+fh(Y,\xi))\\&-\eta(Y)(\phi U^0\nabla_X\xi+\alpha U^0X+(\beta+1)\phi U^0X+fh(X,\xi))\end{aligned} \tag{6.8}$$

for all $X,Y\in D^1\oplus\{\xi\}$.

Proof. Let $X,Y\in D^1\oplus\{\xi\}$. If $D^1\oplus\{\xi\}$ is integrable, then (6.9) is true and from (6.8), we get

$$\begin{aligned}&2F(\nabla_Y\phi X-\nabla_X\phi Y)+2\phi(h(Y,\phi X)-h(X,\phi Y)\\&\quad+\eta(Y)U^0\nabla_X\xi-\eta(X)U^0\nabla_Y\xi+\eta(Y)h(X,\xi)-\eta(X)h(Y,\xi)\\&\quad-\alpha(-\eta(X)FY+\eta(Y)FX)-(\beta+1)(\eta(Y)X-\eta(X)Y)=0.\end{aligned}$$

Applying ϕ to the above equation,

$$\begin{aligned}&-2U^0(\nabla_Y\phi X-\nabla_X\phi Y)+2(h(Y,\phi X)-h(X,\phi Y)\\&\quad+\eta(Y)\phi U^0\nabla_X\xi+\eta(Y)th(X,\xi)+\eta(Y)fh(X,\xi)\\&\quad-\eta(X)th(Y,\xi)-\eta(X)fh(Y,\xi)+\alpha(\eta(X)U^0Y+\eta(Y)U^0X)\\&\quad-(\beta+1)\phi(\eta(X)U^0Y+\eta(Y)U^0X)-\eta(X)\phi U^0\nabla_Y\xi=0.\end{aligned}$$

Hence taking the normal part, we get (6.8). Conversely, let (6.7) and (6.8) be true. Using (6.8) in (6.6),we get

$$\begin{aligned}&-2U^0[X,Y]+2F(\nabla_Y\phi X-\nabla_X\phi Y)+2\phi h(Y,\phi X)\\&\quad-h(X,\phi Y)+\eta(Y)U^0\nabla_X\xi-\eta(X)U^0\nabla_Y\xi+\eta(Y)h(X,\xi)-\eta(X)h(Y,\xi)\\&\quad+\alpha(-\eta(X)FY-\eta(Y)FX)-(\beta+1)(\eta(Y)X-\eta(X)Y)=0.\end{aligned}$$

Applying ϕ to the above equation and using (6.8), we get $\phi U^0[X,Y]=0$, from which we get $U^0[X,Y]=0$. Hence $D^1\oplus\{\xi\}$ is integrable. □

If $\bar{M}$ is a trans-Sasakian manifold, then for all $X \in D^1 \oplus \{\xi\}$ it is know that $h(X,\xi) = 0$ and $U^0\nabla_X\xi = 0$. Hence in view of previous theorem, we have

Corollary 6.6. *If M is a semi-invariant submanifold of a trans-Sasakian manifold with semi-symmetric non-metric connection, then the distribution $D^1 \oplus \{\xi\}$ is integrable if and only if for all $X, Y \in D^1 \oplus \{\xi\}$*

$$h(X, \phi Y) = h(Y, \phi X).$$

7. Integrability of the distribution $\mathbf{D^0} \oplus \{\xi\}$

Lemma 7.1. *Let M be a semi-invariant submanifold of a nearly trans-Sasakian manifold with semi-symmetric non-metric connection. Then*

$$3(A_{FX}Y - A_{FY}X) = P[X,Y] - (\beta + 1)(\eta(Y)PX + \eta(X)PY) \tag{7.1}$$

for all $X, Y \in D^0 \oplus \{\xi\}$.

Proof. For $X, Y \in D^0 \oplus \{\xi\}$ and $Z \in TM$, we have

$$-A_{\phi X}Z + \nabla^{\perp}_Z \phi X = \bar{\nabla}_Z \phi X = (\bar{\nabla}_Z \phi)X + \phi(\bar{\nabla}_Z)X.$$

Using Equation (2.5) we get

$$\begin{aligned} -A_{\phi X}Z + \nabla^{\perp}_Z \phi X = &- (\bar{\nabla}_X \phi)Z + \alpha(2g(X,Z)\xi - \eta(X)Z - \eta(Z)X) \\ &+ (\beta + 1)(\eta(X)\phi Z + \eta(Z)\phi X) \\ &+ \phi\bar{\nabla}_Z X + \phi h(Z,X), \end{aligned}$$

so that

$$\begin{aligned} \phi h(Z,X) = &- A_{\phi X}Z + \nabla^{\perp}_Z \phi X + (\bar{\nabla}_X \phi)Z - \alpha(2g(X,Z)\xi - \eta(X)Z - \eta(Z)X) \\ &+ (\beta + 1)(-\eta(X)\phi Z - \eta(Z)\phi X) - \phi\bar{\nabla}_Z X \end{aligned}$$

and hence we have

$$g(\phi h(Z,X), Y) = -g(A_{\phi X}Y, Z) - g((\bar{\nabla}_X \phi)Y, Z).$$

On the other hand

$$g(\phi h(Z,X), Y) = -g(h(Z,X), \phi Y) = -g(A_{\phi Y}X, Z).$$

Thus from above two relations, we get

$$g(A_{\phi Y}X, Z) = g(A_{\phi X}Y, Z) + g((\bar{\nabla}_X \phi)Y, Z). \tag{7.2}$$

For $X, Y \in D^0 \oplus \{\xi\}$, we calculate $(\bar{\nabla}_X \phi)Y$ as follows. In view of

$$\bar{\nabla}_X \phi Y - \bar{\nabla}_Y \phi X = A_{\phi X}Y - A_{\phi Y}X + \nabla^{\perp}_X \phi Y - \nabla^{\perp}_Y \phi X$$

and

$$\bar{\nabla}_X \phi Y - \bar{\nabla}_Y \phi X = (\bar{\nabla}_X \phi)Y - (\bar{\nabla}_Y \phi)X + \phi[X,Y],$$

we have

$$(\bar{\nabla}_X \phi)Y - (\bar{\nabla}_Y \phi)X = A_{\phi X}Y - A_{\phi Y}X + \nabla^{\perp}_X \phi Y - \nabla^{\perp}_Y \phi X - \phi[X,Y],$$

which in view of (2.5) gives

$$
\begin{aligned}
(\bar{\nabla}_X\phi)Y =& \frac{1}{2}(A_{\phi X}Y - A_{\phi Y}X + \nabla^{\perp}_X\phi Y - \nabla^{\perp}_Y\phi X - \phi[X,Y]) \\
&+ \frac{\alpha}{2}(2g(X,Y)\xi - \eta(X)Y - \eta(Y)X) \\
&- \frac{(\beta+1)}{2}(\eta(X)\phi Y + \eta(Y)\phi X).
\end{aligned} \tag{7.1}
$$

Now using (7.3) in (7.2) we get (7.1). □

In view of $Ker(P) = D^0 \oplus \{\xi\}$, this leads to the following.

Theorem 7.2. *Let M be semi-invariant submanifold of a nearly trans-Sasakian manifold with semi-symmetric non-metric connection. Then the distribution $D^0 \oplus \{\xi\}$ is integrable if and only if*

$$A_{FX}Y = A_{FY}X$$

for $X, Y \in D^0 \oplus \{\xi\}$.

Using (2.2) in (7.2) for $X, Y \in D^0 \oplus \{\xi\}$, we get $A_{FX}Y = A_{FY}X$. Hence in view of the above theorem, we get the following.

Corollary 7.3. *Let M be a semi-invariant submanifold of a trans-Sasakian manifold with semi-symmetric non-metric connection. Then the distribution $D^0 \oplus \{\xi\}$ is integrable.*

8. Integrability of the distribution $\mathbf{D^0}$ and $\mathbf{D^1}$

Since we calculated the torsion tensor $N^1(Y, X)$ for $Y, X \in D^0$, it can be verified that

$$
\begin{aligned}
\phi((\bar{\nabla}_X\phi)Y - (\bar{\nabla}_Y\phi)X) =& \phi(A_{\phi X}Y - A_{\phi Y}X) + \phi(\nabla^{\perp}_X\phi Y - \nabla^{\perp}_Y\phi X) \\
&+ [X,Y] - \eta([X,Y])\xi,
\end{aligned} \tag{8.1}
$$

and

$$
\begin{aligned}
(\bar{\nabla}_{\phi Y}\phi)X - (\bar{\nabla}_{\phi X}\phi)Y =& [X,Y] + \phi(A_{\phi X}Y - A_{\phi Y}X) \\
&+ \phi(\nabla^{\perp}_X\phi Y - \nabla^{\perp}_Y\phi X).
\end{aligned} \tag{8.2}
$$

Using (8.1), (8.2) and (7.1), we get

$$N^1(Y,X) = \frac{8}{3}[X,Y] + \frac{2}{3}\phi(\nabla^{\perp}_X\phi Y - \nabla^{\perp}_Y\phi X) + \frac{8}{3}d\eta(X,Y)\xi \tag{8.3}$$

for $Y, X \in D^0$.

Theorem 8.1. *The distribution D^0 is integrable on a semi-invariant submanifold M of a nearly trans-Sasakian manifold with semi-symmetric non-metric connection if and only if*

$$N^1(Y,X) \in D^0 \oplus \bar{D}^1, \tag{8.4}$$

$$A_{FX}Y = A_{FY}X, \tag{8.5}$$

where $X, Y \in D^0$.

Proof. If D^0 is integrable, then in view of (8.2) and (8.3), the relation (8.4) and (8.5) follow easily. Conversely, Let $X, Y \in D^0$ and let the relation (8.4) and

(8.5) be true. Then in view of (8.2), we get $P[X, Y] = 0$. In view of (8.3) we get $g(\xi, N^1(Y, X)) = g(\xi, 2[Y, X]) = 0$. Thus $[X, Y] \in D^0$. □

Theorem 8.2. *Let M be a semi-invariant submanifold of a nearly trans-Sasakian manifold with semi-symmetric non-metric connection with $\alpha \neq 0$. Then the non-zero invariant distribution D^1 is not integrable.*

Proof. If D^1 is integrable, then for $X, Y \in D^1$ it follows that $d\eta(X, Y) = 0$ and $[\phi, \phi](X, Y) \in D^1$. Therefore, for $X \in D^1$ in view of (4.5) we get

$$\eta([\phi, \phi](X, PX) + 2d\eta(X, PX)\xi) = 0,$$

$$\eta(N^1(X, PX)) = 4\alpha g(\phi X, PX) = 4\alpha g(PX, PX),$$

which is a contradiction. □

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Mobin Ahmad,
Department of Mathematics,
Faculty of Science,
Jazan University, Jazan
Kingdom of Saudi Arabia.
E-mail address: mobinahmad@rediffmail.com

Jae-Bok Jun,
Department of Mathematics,
College of Natural Science,
Kookmin University,
Seoul 136-702, KOREA
E-mail address: jbjun@kookmin.ac.kr

Mohd Danish Siddiqi,
Department of Pure Mathematics,
Faculty of Science,
Jazan University, Jazan,
Kingdom of Saudi Arabia.
E-mail address: anallali@yahoo.com